Price Impact of Trades for Cryptocurrencies

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1 Introduction

Price impact, as the name suggests, refers to the impact that a trade has on the price of the object being traded. This has been an active area of interest for researchers, and there are many models and empirical studies that have been performed. However, these studies have typically looked at data from 'standard' stocks, and so a natural question to ask is whether these models and empirical result continue to hold for cryptocurrencies, which are of a seemingly different nature.

The most common way to answer this question and to quantify the effect of a trade is through the *response function* $\mathcal{R}(l, V)$, where the arguments l and V are the time after and the volume of the trade, respectively. Denote the price at time n by p_n and the corresponding volume V_n, and represent whether the trade at time n was a buy or a sell by the variable $\varepsilon_n = \pm 1$; let $\varepsilon_n = 1$ signify a buy and $\varepsilon_n = -1$ signify a sell. With these definitions, the response function is defined as [1]

$$\mathcal{R}(l, V) = \mathbb{E}((p_{n+l} - p_n)\varepsilon_n \mid V_n = V), \tag{1}$$

where $\mathbb{E}(\cdot)$ denotes the expected value (average). That is, for a trade at time n and of volume V_n, the response function $\mathcal{R}(l, V)$ is the average change in price after a time of l has elapsed, conditioned on the fact that $V_n = V$. Models of price impact hence seek to find theoretical expressions for (1).

This paper will present some initial empirical investigations into how price impact models apply to cryptocurrencies by examining trading data. Trading data was obtained from the exchange Bitfinex, for Bitcoin (BTC), Ethereum (ETC), Monero (XMR) and ZCash (ZEC). Trading data was taken starting from the beginning of December, and ends either after 5×10^5 trades (BTC and ETH) or at the end of December (XMR and ZEC, approximately 4.6×10^5 and 6.0×10^5 trades, respectively).

2 Models

There are a variety of models for the price impact function (1), for which a comprehensive survey has been done by Bouchaud [2]. One of the simplest and earliest models is Kyle model [3]

2.1 Kyle Model

In the Kyle model, the impact is linear in the volume traded and is permanent in time. That is, for some measure of impact λ we can loosely say that

$$\mathcal{R}(l, V) = \lambda \varepsilon V.$$

Additionally, the price change between $t = 0$ and $t = T$ (corresponding to N trades, say) will be determined by [4].

$$p_T - p_0 = \lambda \sum_{n=0}^{N-1} \varepsilon_n V_n.$$

But if price change is to behave like a random walk, then the signs of ε_n should be serially uncorrelated. If this were the case, then we should find that the quantity

$$\mathcal{C}_0(l) = \mathbb{E}(\varepsilon_{n+l}\varepsilon_n) - \mathbb{E}(\varepsilon_n)^2$$

quickly goes to zero after a few trades [1]. However, this doesn't seem to be the case; for BTC we find that rather the proposed power law

$$\mathcal{C}_0(l) = \frac{C_0}{l^\gamma}$$

seems to hold, as can be seen in Fig. 1. Additionally, empirical evidence for many stocks suggests that for shorter time periods the scaling is sublinear in volume [4], which again is counter to the Kyle model. In reality, it seems that the impact function is both nonlinear as well as transient (not permanent in time).

2.2 Transient Model

One of the ingredients in the development of the transient model is the fact that the impact function can be factorised into a volume dependent part and a time dependent part:

$$\mathcal{R}(l, V) = \mathcal{R}(l) f(V),$$

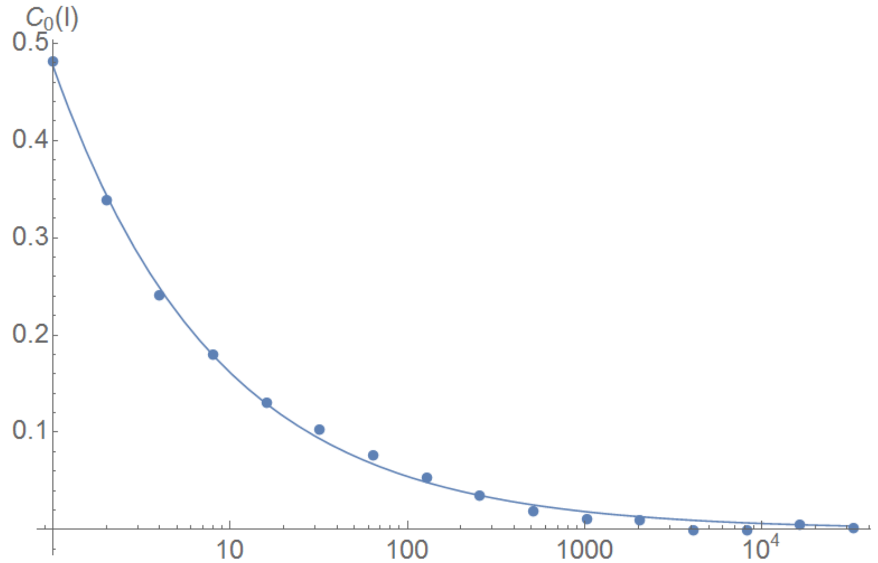

Figure 1: Empirical values of $\mathcal{C}_0(l)$ (blue dots) and the fitted power law (blue curve) for BTC. The fitting parameters are $C_0 \approx 0.477$ and $\gamma \approx 0.471$.

where empirically it seems that $f(V) \propto \ln(V)$ [5]. From this, one need only characterise $\mathcal{R}(l)$ to find the impact function. The result is that [1]

$$\mathcal{R}(l) = \mathbb{E}(\ln(V_n))G_0(l) + \sum_{n=1}^{l-1} G_0(l-n)\mathcal{C}_1(n) + \sum_{n>0} (G_0(l+n) - G_0(n))\, \mathcal{C}_1(n),$$

where

$$\mathcal{C}_1(n) = \mathbb{E}(\varepsilon_{n+l}\varepsilon_n \ln(V_n)),$$

and G_0 is a decay factor that determines how the instantaneous impact of a trade decays over time. The decay factor is modelled as

$$G_0(l) = \frac{\Gamma_0 l_0^\beta}{(l_0 + l)^\beta},$$

or the similar form (which matches well with empirical data [2])

$$G_0(l) = \frac{\Gamma_0}{(l_0^2 + l^2)^{\frac{\beta}{2}}}.$$

The result is that after fitting the model properly, we obtain an impact function that initially increases to a peak and then decays to zero. Thus, if the model holds we expect to see a similar shape in the empirical values of $\mathcal{R}(l, V)$.

3 Results

Empirical data for BTC, ETH, XMR and ZEC trades on the Bitfinex exchange were used to compute $\mathcal{R}(l, V)$. The trading data did not include the price of the currency at the time of the trade, so this was approximated using the price point of the trade itself. The computation was performed based on the definition (1). The empirical values for BTC, ETC, XMR and ZEC are shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 respectively. To condition on a volume, only data for a given range of volumes was used in the computation. Ranges where chosen to maximise the amount of data in the range. The ranges were $[0.5, 1)$, $[1, 2)$, $[0.2, 0.3)$ and $[0.2, 0.3)$ for BTC, ETH, XMR and ZEC respectively.

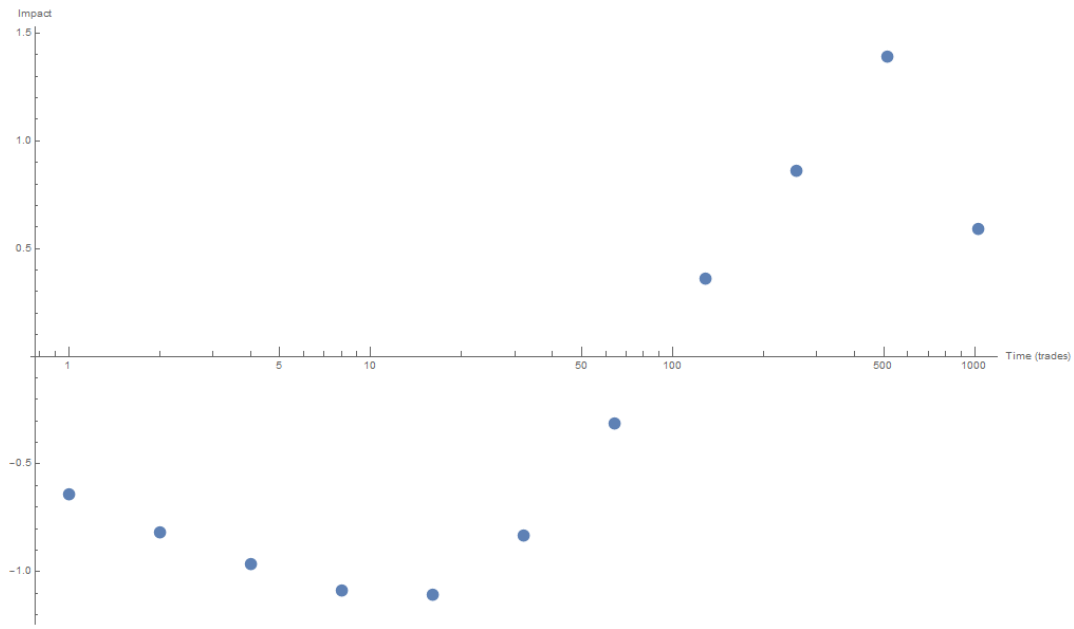

Figure 2: Empirical values of $\mathcal{R}(l, V)$ for BTC trading data.

From the figures, it can be seen that the empirical impact functions have approximately the right shape for the transient model. One interesting feature of the graphs that was not predicted by the model is the initial decrease to a negative minimum before the increase to the positive peak. One possible explanation to account for this comes from the dynamics of the order book and the fact that the price was approximated using the price point of the executed order: when a sell is executed, it is most likely to be executed at the lowest available sell point, since this is the best deal for a buyer. This removes the lowest sell from the order book, and so the next buy will have to be executed on a sell at a higher price point, and so the next sell will generally be at a higher price point which corresponds to the price going up. This will contribute to the initial decrease seen in the figures, and will be counteracted after the order book receives more orders to replace the ones executed. Conversely, for a buy, similar logic leads us to the conclusion that buys at the higher price points will be executed first, as this is the best deal for the seller, which in turn will cause the price to go down.

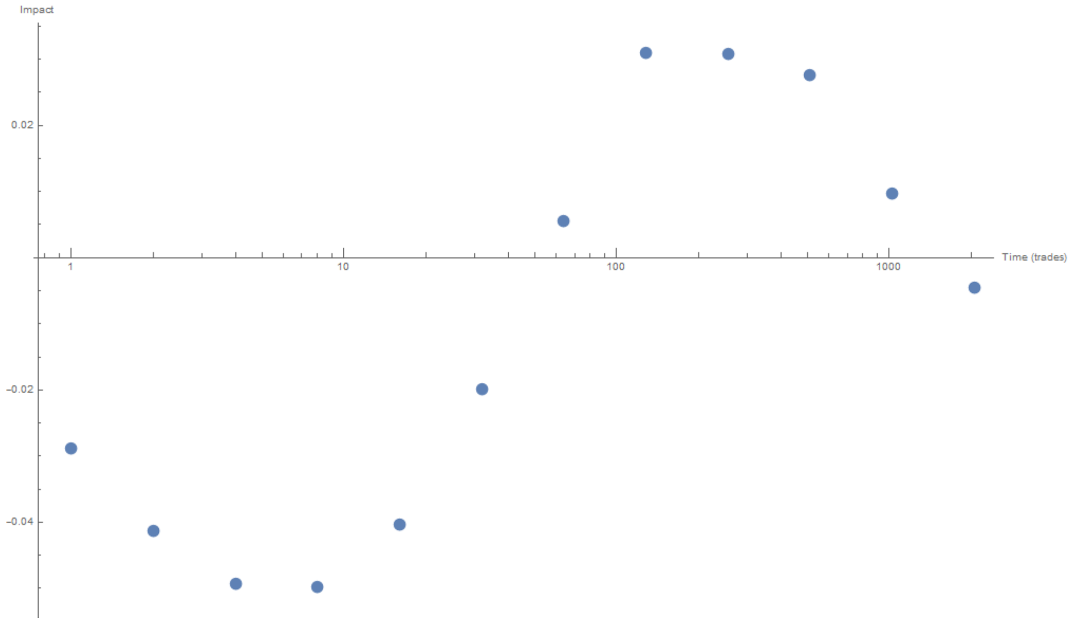

Figure 3: Empirical values of $\mathcal{R}(l, V)$ for ETH trading data.

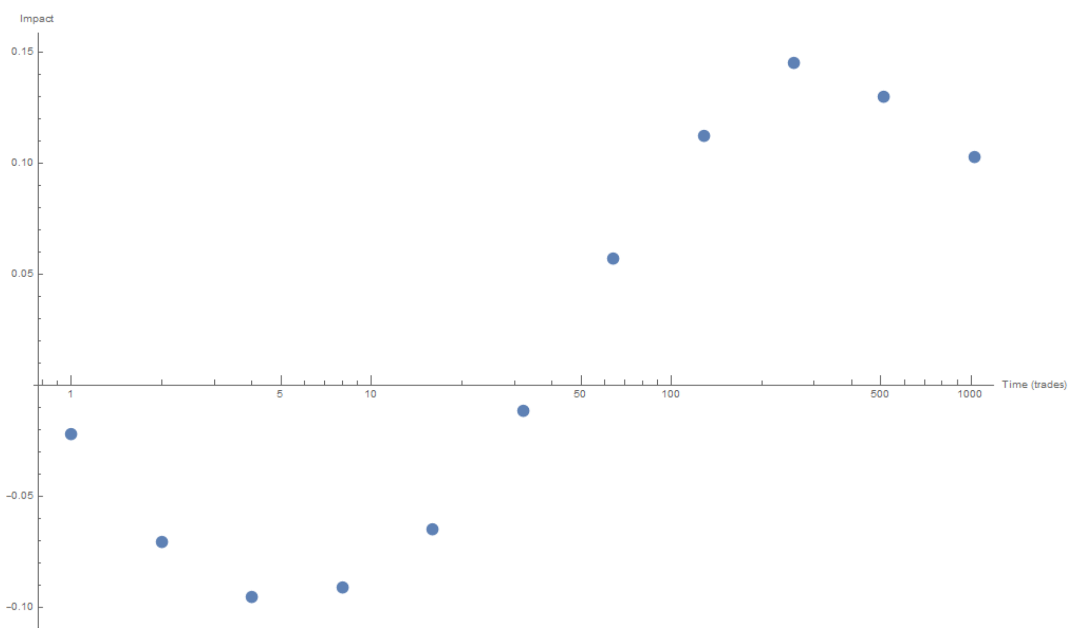

Figure 4: Empirical values of $\mathcal{R}(l, V)$ for XMR trading data.

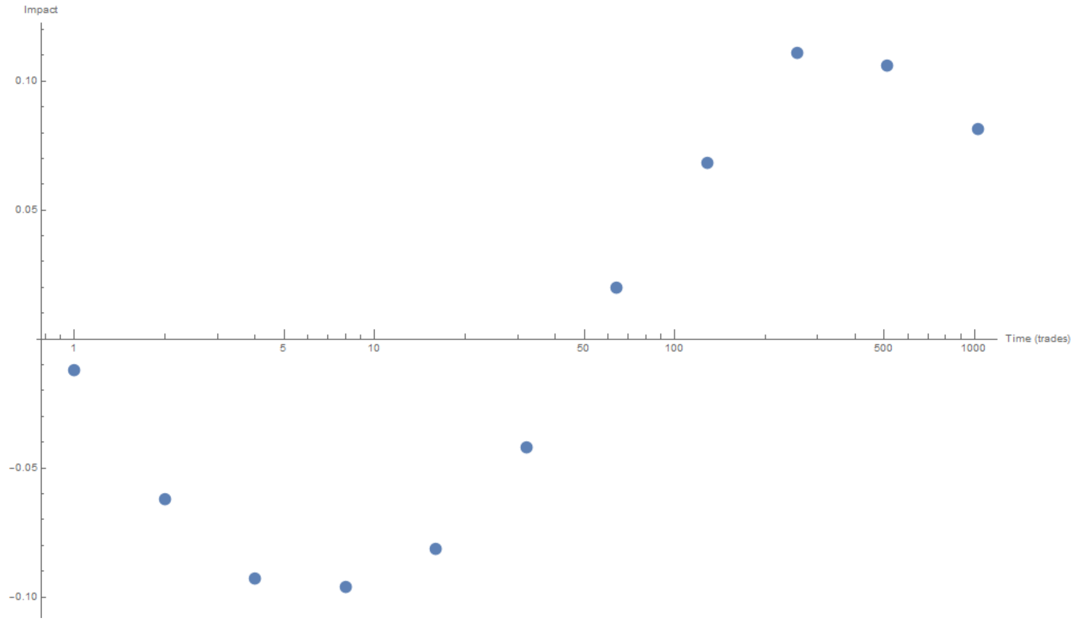

Figure 5: Empirical values of $\mathcal{R}(l, V)$ for ZEC trading data.

4 Conclusions and Future Work

The goal of this paper was to investigate the impact that trades have on the prices of cryptocurrencies. We found that Current models that are used for 'standard' stocks seem to apply to cryptocurrenices, with the key result being that price impact in this realm does not seem to be permanent. The general shape of the curve is characterised by an initial increase to a peak, after which impact decays to zero.

Based on these results and using the gathered data, further investigations are possible. One interesting avenue to pursue is to find the fitting parameters for some of the functions constituting the models, as they can actually indicate whether arbitrage is possible [6].

References

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